

The Cluster-Span Threshold: network binarisation for information-rich medium density ranges

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Abstract—Objective: Research into network modelling of brain function faces a methodological challenge in selecting an appropriate threshold to binarise edge weights. We present a novel, non-arbitrary threshold for functional connectivity brain networks named the Cluster-Span Threshold (CST) which ensures a trade-off of sparsity and information richness in the network.

Methods: We compare CST networks with weighted networks, minimum spanning trees, union of shortest path graphs and proportional arbitrary thresholds. We test these techniques on weighted complex hierarchy models by contrasting models with small parametric differences via *t*-tests of network metrics. Using this comparison approach we also test the robustness of the techniques to alterations in the network in the form of random and targeted attacks. We extend these results with the analysis of three relevant EEG datasets: eyes open and closed resting states; visual short-term memory tasks; and resting state Alzheimer’s disease with a healthy control group.

Results: We find that the CST consistently outperforms other state-of-the-art binarisation methods for topological accuracy and robustness. These results are confirmed in the real data, where the CST proves the most sensitive of non-arbitrary techniques.

Conclusion: The CST is a sensitive, multi-purpose threshold to aid network modelling of functional connectivity.

Significance: We provide a non-arbitrary and robust solution to the problem of binarising functional networks- a key processing step for functional network analysis. We also provide insights into the effects of network size and density on the topological accuracy of a binarised functional connectivity network.

I. INTRODUCTION

Network analysis of functional connectivity is an established framework for extracting functional information from the brain recorded using various platforms, most prominently the Electroencephalogram (EEG) and the Magnetoencephalogram (MEG) for high temporal resolution and functional Magnetic Resonance Imaging (fMRI) for high spatial resolution [1]. Networks excel in their ability to capture the interdependent activity which underlies brain function [2], enabling

powerful methods for classification of clinical data [3] including Alzheimer’s disease (AD) [4] and Schizophrenia [5]. However, connectivity defined between all possible pairs of brain regions, whether sensors or cortical sources from the EEG or the MEG or partitioning of spatial regions in fMRI, present the researchers with a stifling amount of information of which a large portion can be regarded as spurious [2]. To resolve these issues, a compact binarised form of the network is generally sought after which explains the main topology of the underlying activity. Such a method avoids problems of weight normalisation and allows for an easy comparison of different pre-processing choices [3]. Selecting a method to binarise the network is thus seen as a major step in network construction in which the researcher is presented with a large degree of subjective choice [2], [6], [7]. We present here a full investigation of a novel methodology to threshold networks, named the Cluster-Span Threshold (CST), significantly extending preliminary results in [8] and [9], and compare it with recently proposed alternatives in the field.

Recent research emphasises the importance of solutions to the thresholding or binarisation problem in functional connectivity [10]–[15]. While some find sparsity desirable [12], others found higher densities to be as or more relevant [15]–[17]. Using simulations, we seek to clarify how network size and density range may effect the ability to discern small topological differences in network topology. Other researchers look towards the integration of different density ranges, however such an approach will have a tendency towards diluting the potency of potential differences [10] or falling prey to the multiple comparison problem.

The CST offers a novel approach for binarisation and appears to be the only solution as yet which lies with consistency within medium density ranges (30-50%). A trade-off of integrative and segregative network behaviour is known to allow the brain to function at a highly efficient rate [18]. The CST is thus defined at the point of balance of integrative and segregative behaviour, measured by the clustering of the topology, of the connected network. The networks can be analysed to reveal how the topologies are composed at this equilibrated state.

In preliminary analysis, the CST has been shown to outperform both the Minimum Spanning Tree (MST) [12], [19] and the Union of Shortest Paths (USP) [14] in separate studies for distinguishing subtly different topologies [8], [9]. The CST and MST were compared in their ability to distinguish topological differences between a set of cognitive tasks performed by healthy young adults which are relevant to the sensitive and specific detection of AD [8]. The CST and USP were com-

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pared in a clinical study of networks formed from beta activity in a population of AD patients alongside age matched healthy control subjects [9]. Here we provide an extensive and full comparison of the CST with the MST, USP, weighted metrics and a number of arbitrary density thresholds for distinguishing differences in simulated EEG networks introduced by Smith & Escudero in [20]. We further analyse these techniques when the simulations are subject to random and targeted hub attacks on the network [21] to test their robustness in representing true network characteristics in the face of noise and/or outliers in the estimation of coupling between brain time series. We go on to apply our non-arbitrary binarisation techniques to three real EEG datasets. We compare our thresholds on distinguishing the well known alpha activity existing between eyes open vs eyes closed resting state conditions in healthy volunteers with a 129 channel EEG [27]. We then compare these techniques for distinguishing visual short-term memory binding tasks in healthy young volunteers with a 30 channel EEG [8]. Finally, we compare our techniques in distinguishing between AD patients and healthy control in a 16 channel EEG set-up [29]. The varying sizes of these networks avoids any potential bias of network size on the network binarisation methods in real applications.

II. METHODS

This section details the binarisation techniques, network metrics, network simulations and real datasets used in this study. Let \mathbf{W} be a weighted adjacency matrix for an undirected graph $G = (\mathcal{E}, \mathcal{V})$ such that w_{ij} is the weight of the edge between vertices i and j in G and G has no multiple edges or self-loops. A simple graph is a graph such that $w_{ij} \in \{0, 1\}$ for all i and j and $w_{ij} = 0$ for $i = j$, where 1 indicates the existence of an edge. Then $n = |\mathcal{V}|$ is the size of the network and m is the number of undirected edges so that $2m$ is the number of non-zero entries in the symmetric adjacency matrix.

A. Methods for Network Binarisation

1) *Cluster-Span Threshold*: The CST chooses the binary network at the point where open to closed triples are balanced [8]. This balance occurs when $C_{Glob} = 0.5$, which is obvious from the definition. Importantly, the balancing of this topological characteristic necessarily endows the binary network with a trade off of sparsity to richness of information. We see this since a network is dense if most triples are closed and sparse if most triples are open.

The algorithm for the CST computes the binary networks for each possible number of strongest edges between 15% to 85%, rounded to the closest real edge density, i.e., for a network with n nodes, each binary network of the M strongest edges from

$$Y = \operatorname{argmin}_m \left\{ \left| \frac{2m}{n(n-1)} - 0.85 * n(n-1) \right| \right\} \quad (1)$$

down to

$$X = \operatorname{argmin}_m \left\{ \left| \frac{2m}{n(n-1)} - 0.15 * n(n-1) \right| \right\}. \quad (2)$$

The clustering coefficient is then computed for each of these networks, obtaining a vector

$$\mathbf{C} = \{C_Y, C_{Y-1}, \dots, C_{X+1}, C_X\}. \quad (3)$$

Then the network of the CST is the binary network corresponding to the equation

$$Z = \operatorname{argmin}_i (C_i - 0.5), \quad (4)$$

i.e. the threshold achieving minimum value of the vector \mathbf{C} minus the clustering coefficient value which obtains an equilibrium between triangles and non-triangle triples, 0.5. The values of 15% and 85% are chosen as safe values based on experimental evidence [8], [9]. Particularly, below 15%, real brain networks can have a tendency to fracture into more than one component, thus making calculations of metric values, including the clustering coefficient, inconsistent and unreliable [16].

Similarly to arbitrary edge density thresholds, the CST is robust to variable weight connectivities as it is based on topological features void of edge weights. By analysing the connection densities at which the CST of different networks inhabits, one can gather other required topological information of the networks. For example, scale-freeness and levels of integration in the CST network can be obtained by comparing with simulated scale-free and random networks.

2) *Arbitrary Threshold*: Arbitrary thresholds can be chosen by either choosing a weight above which edges are kept and below which edges are discarded, or by choosing a percentage of strongest weighted edges to keep in the network. The latter choice is more robust and easier to compare between different set-ups and subjects because it keeps the connection density constant and thus is not affected by the values of the weights, which may vary wildly particularly when considering the comparison of different connectivity measures. Thus, here we choose arbitrary thresholds at 10%, 20%, 30%, 40%, and 50% to make sure we cover a wide array of connection densities whilst reducing redundancy. Similarly as for the CST, for the threshold at $T\%$ this corresponds to binary network corresponding to the number of strongest edges X , where

$$X_T = \operatorname{argmin}_m \left\{ \left| \frac{2m}{n(n-1)} - T * n(n-1) \right| \right\}. \quad (5)$$

3) *Minimum Spanning Tree*: The MST is a construction based binarisation approach which obtains a tree by selecting the strongest edges of the network such that the network is fully connected and no cycles are present. The algorithm for its construction is well known [22] and included in popular toolboxes [23].

4) *Union of Shortest Path Lengths*: The USP is another construction based approach to unbiased network binarisation [14]. The shortest path between two nodes in a network is the set of edges with the minimum sum of weights connecting them. This can be constructed using e.g. Dijkstra's [24] algorithm to find the shortest paths between each pair of nodes in the network, adding all the edges of those paths to an initially empty binary network. Because connectivity has an inverse relation to distance, the weights of the network must

first be relationally inversed in order to construct the shortest paths. This inversion process can take several forms which involves a certain amount of subjective discretion and depends largely on the distribution of the original weights. For our study we choose $\tilde{\mathbf{W}} = -\ln(\mathbf{W})/\alpha$, where

$$\alpha = \min\{\mathbb{N}\} \text{ s.t. } \max_{i,j}(\hat{w}_{ij}) < 1, \quad (6)$$

as it has been shown to offer a better spread of metric magnitudes which is important for shortest path problems [9].

B. Network Metrics

To analyse the simulated and real EEG networks we use a variety of common metrics.

- The **characteristic path length**, L , is the average of the shortest path lengths between all pairs of nodes in the graph [25].
- The **efficiency**, Eff of a weighted network is the mean of one over the shortest path lengths, thus inversely related to L [23].
- The **diameter**, D , of a graph is the largest shortest path length between any two nodes in the graph [12].
- The **clustering coefficient**, C , is the mean over i of the ratio of triangles to triples centred at node i [25].
- The **weighted clustering coefficient**, C_W , is a weighted version of the clustering co-efficient for binary networks.
- The **leaf fraction**, LF , of a tree is the fraction of nodes in the graph with degree one. Note, every path containing such a node either begins or ends at that node [12].
- The **edge density**, P , is the ratio of the number of edges in the graph to the total possible number of edges for a graph with the same number of nodes, i.e. $P = 2m/n(n-1)$ [25]. For the CST, P takes an inversely relational position to C of proportional thresholds. This can be seen by considering two weighted networks whose values of C increase monotonically with increasing P and such that one has higher values of C than the other, which is a working assumption in our case. Then the network with the greater values of C will attain its CST at a lower density, P . In a similar vein, P of the USP is inversely related to L of proportional thresholds- the higher the density of the USP, the shorter the average shortest path in the weighted network.
- The **degree variance**, V , is the variance of the degrees of the graph which distinguishes the level of scale-freeness present in the graph topology [20].
- The **maximum degree**, MD , of a network is just as named- the degree of the node with the most adjacent edges in the network [12].

For each binarisation technique we choose three metrics to analyse the subsequent binary networks. These differ for each technique because of the construction of the network. Particularly, the MST metrics are chosen based on a study of Tewarie et al. [12]. Similarly, we choose three weighted metrics for analysing the original weighted networks. These choices can be found in Table I. We try as much as possible to stick to three main categories of metrics for each binarisation technique: segregation (M1 in Table I), efficiency (M2) and

irregularity (M3) [1], [20]. This notably deviates for M3 in the weighted case where the mean weight of the network edges, μ_W , is an appropriate and more obvious choice of metric than the variance of those weights.

TABLE I
GROUPED TOPOLOGICAL METRICS- THREE FOR EACH NETWORK TYPE

Metric	CST	MST	USP	Weight	%T
M1	P	LF	C	C_W	C
M2	L	D	P	Eff	L
M3	V	MD	V	μ_W	V

C. Network Simulations

We use simulations of networks to test the various binarisation techniques for the factors of varying hierarchical topology and network size. This is implemented using the Weighted Complex Hierarchy (WCH) model with different strength parameters [20]. This model allows fine tuning of topology and simulates the topological characteristics of EEG phase-lag networks. By selecting parameters with small differences we can probe the ability of these binarisation techniques to discern small differences in topology of populations.

The WCH network takes the existence probabilities of edges in an Erdős-Rényi random network [26] as the base weights of the edges. It then randomly chooses the number of levels of the hierarchy (between 2 and 5) before randomly selecting nodes, based on the probability that nodes exist in a given level of the hierarchy, to which an arbitrarily chosen additional weight, the strength parameter, is added to all adjacent edges. By fine-tuning the strength parameter, these networks have been shown to mimic the topology of EEG networks formed from the weighted phase-lag index [20].

We repeat this methodology for networks with 16, 32, 64 and 128 nodes, spanning a large range of network sizes as used in current research, e.g. see [4].

To test the robustness of the given techniques, we subject simulated networks to random and targeted attacks before implementing similar topological comparative analysis as with the original networks. Random attacks are implemented by substituting the models weights, with the sparse, randomly weighted adjacency matrix entries where those entries are non-zero. We implement this comparison by increasing the density of the sparse matrix, i.e. densities of 0, 0.05, 0.1, ..., 0.95, 1. Targeted attacks are implemented similarly to random attacks except the attacks are restricted only to those nodes whose average adjacent weight is over one standard deviation above the mean, relating to those nodes with abnormally strong connectivity. Such strongly weighted nodes are known as 'hub' nodes for their importance to the topology of the network.

D. Data acquisition and pre-processing

1) *Eyes open - Eyes closed resting state*: The eyes-open, eyes closed 129 node dataset is available online under an Open Database License. We obtained the dataset from the Neurophysiological Biomarker Toolbox tutorial [27]. It consists for 16 volunteers and is downsampled to 200Hz. We used the clean dataset which we re-referenced to an average reference before further analysis.

2) *Visual Short-Term Memory Binding tasks*: We study a 30-channel EEG dataset for 19 healthy young volunteers participating in different VSTM tasks. Full details of the task can be found in [17]. Written consent was given by all subjects and the study was approved by the Psychology Research Ethics Committee, University of Edinburgh. The task was to remember objects consisting of either black shapes (Shape) or shapes with associated colours (Binding), presented in either the left or right side of the screen. The sampling rate is 250 Hz and a bandpass of 0.01-40 Hz was used in recording.

3) *Alzheimer's disease*: The EEG recordings were taken from 12 AD patients and 11 healthy control subjects. The patients -5 men and 7 women; age = 72.8 ± 8.0 years, mean \pm standard deviation (SD)- were recruited from the Alzheimer's Patients' Relatives Association of Valladolid (AFAVA). They all fulfilled the criteria for probable AD. EEG activity was recorded at the University Hospital of Valladolid (Spain) after the patients had undergone clinical evaluation including clinical history, neurological and physical examinations, brain scans and a Mini Mental State Examination (MMSE) to assess their cognitive ability [28]. The local ethics committee approved the study and control subjects and all caregivers of the patients gave their informed consent for participation. The 16 channel EEG recordings were made using Profile Study Room 2.3.411 EEG equipment (Oxford Instruments) in accordance with the international 10-20 system. Full details can be found in [29].

4) *Functional connectivity*: For each dataset, FIR bandpass filters were implemented for α (8-13Hz), β (13-32Hz) or both at the specified sampling frequency using Hamming windows. We use a filter order of 70 to provide a good trade-off between sharp transitions between the pass and stop bands while keeping the filter order low. The filtered signals were then analysed for pairwise connectivity using the Phase Lag Index (PLI) [30]. The PLI between time series i and j is defined as

$$PLI_{ij} = |\langle \text{sgn}(\phi_i(t) - \phi_j(t)) \rangle|, \quad (7)$$

where the instantaneous phase at time t , $\phi_k(t)$, is regarded as the angle of the Hilbert transform of the signal k at time t . These values were averaged over trials to obtain one connectivity matrix per subject per condition.

III. RESULTS

A. Sensitivity to subtle topological differences

We undergo 50 simulated trials of two populations of 20 networks selected from banks of 1000 WCH networks with strength parameters $s = 0, 0.05, 0.1, 0.15, 0.2, 0.25$ and 0.3 . We binarise these networks using each of our binarisation methods. We then compute the three metrics, M1, M2 and M3, for each of these networks (weighted metrics are computed from the original weighted networks). We perform population t -tests of these metrics for WCH binarisations with strength parameters with a difference of 0.05, i.e. $s = 0$ vs. $s = 0.05$, $s = 0.05$ vs. $s = 0.1$, $s = 0.1$ vs. $s = 0.15$, ..., $s = 0.25$ vs. $s = 0.3$. We choose the best metric of the three to represent the ability of the binarisation method to discern

subtle topological differences where the 'best' metric is chosen as that which attains the maximum number of significant p -values less than the standard $\alpha = 0.05$ level. If two or more metrics obtain the maximum value, we then choose the one with the lowest mean log of p -values. Choosing the log in this instance emphasises the importance of smaller p -values for distinguishing differences. We are aware that this is a slight abuse of the p -value, but in this instance we take it as a proxy measure for the ability to discern subtle topological differences in a way which is easily translatable to clinical studies.

Table II shows the results for differences discovered between WCH models with differences in strength parameter of 0.05. The CST is shown to outperform other non-arbitrary methods in general. In testing the comparisons of the WCH model with varying strength parameter, s , it discovers significant differences at the 5% level 71.3% of the time over all conditions (Table II). In comparison, the MST discovers just 22.4% of the differences, the USP discovers 50% of the differences and the weighted metrics discover just 40.5% of the differences. Out of these methods, in fact, it discovers the most differences in all but two conditions- those being the 0.1 vs 0.15 comparison in the 16 node networks and the 0.25 vs 0.3 node comparison in the 128 node cases, of which the USP is best on both occasions.

In comparison with arbitrary percentage thresholds the CST appears to perform approximately the same as the 40% which discerns a slightly higher rate of 71.5% of differences overall. The 50% threshold also appears to be good at discerning differences here with an overall rate of 70.4% of differences discovered. It is important to note that the question we attempt to answer here is "how do non-arbitrary methods compare to a wide range of arbitrary proportional thresholds?", since the arbitrariness itself is the factor we attempt to overcome, rather than simply to find the best possible threshold for these simulations. With this in mind we can see that, compared to the other techniques, the CST far outperforms the field in this study and, indeed, is achieving close to maximal results in comparison with arbitrary thresholds.

B. Robustness to random and targeted attacks

The robustness to random and targeted attacks is evaluated by comparing the metrics from the attacked WCH models over all non-arbitrary techniques using t -tests as before. We look at the case with maximum ratio between the mean and standard deviation over methods, i.e. the case which maximises the ratio of average performance and comparability of performances. This happens in the 32 node case (6.3538) with differences in strength parameter of $s = 0.1$ and $s = 0.15$ (see Table II). The grand percentage over all sizes of attack of p -values below 0.05 for each metric is presented in Tables III and IV for random and targeted attacks, respectively. These show that CST maintains the highest average accuracy of distinguishing topological differences with consistency across metrics.

The maxima of metrics for each method for each size of attack is shown in Fig. 1 and Fig.2 for random and targeted attacks, respectively. These results clearly show the robustness of the CST to even large scale attacks.

TABLE II

PERCENTAGE OF TOPOLOGICAL DIFFERENCES DISCOVERED BETWEEN WEIGHTED COMPLEX HIERARCHY MODELS ($s = x$ vs. $s = y$). BOLD IS THE BEST NON-ARBITRARY TECHNIQUE, UNDERLINED IMPLIES THE ARBITRARY THRESHOLD DOES AS WELL OR BETTER THAN NON-ARBITRARY TECHNIQUES.

16 Nodes	CST	MST	USP	Weight	50% T	40% T	30% T	20% T	10% T
0.1 vs 0.15	58%	42%	74%	62%	64%	66%	70%	<u>76%</u>	34%
0.15 vs 0.2	46%	24%	38%	32%	40%	46%	44%	34%	18%
0.2 vs 0.25	40%	20%	14%	10%	<u>44%</u>	38%	34%	32%	14%
0.25 vs 0.3	24%	12%	16%	18%	<u>26%</u>	<u>24%</u>	<u>24%</u>	18%	14%
Average	42%	25%	36%	31%	<u>44%</u>	<u>44%</u>	<u>43%</u>	40%	20%
32 Nodes	CST	MST	USP	Weight	50% T	40% T	30% T	20% T	10% T
0.1 vs 0.15	90%	66%	66%	70%	90%	96%	96%	96%	78%
0.15 vs 0.2	90%	40%	48%	46%	88%	94%	90%	80%	58%
0.2 vs 0.25	56%	12%	26%	16%	54%	52%	54%	46%	18%
0.25 vs 0.3	38%	6%	24%	12%	38%	44%	34%	18%	28%
Average	69%	31%	41%	36%	68%	<u>72%</u>	<u>69%</u>	60%	46%
64 Nodes	CST	MST	USP	Weight	50% T	40% T	30% T	20% T	10% T
0.1 vs 0.15	100%	52%	76%	80%	98%	100%	98%	100%	92%
0.15 vs 0.2	90%	24%	54%	52%	88%	88%	<u>92%</u>	86%	58%
0.2 vs 0.25	76%	8%	32%	14%	70%	68%	68%	40%	20%
0.25 vs 0.3	72%	12%	48%	24%	<u>72%</u>	<u>72%</u>	50%	42%	20%
Average	85%	24%	53%	42.5%	82%	82%	77%	67%	48%
128 Nodes	CST	MST	USP	Weight	50% T	40% T	30% T	20% T	10% T
0.1 vs 0.15	100%	22%	72%	72%	<u>100%</u>	<u>100%</u>	<u>100%</u>	<u>100%</u>	98%
0.15 vs 0.2	100%	8%	72%	58%	<u>100%</u>	98%	<u>100%</u>	88%	72%
0.2 vs 0.25	98%	4%	58%	26%	86%	86%	84%	68%	26%
0.25 vs 0.3	62%	6%	82%	14%	68%	72%	38%	36%	12%
Average	90%	10%	71%	43%	89%	89%	81%	73%	52%
Grand Average	71.3%	22.4%	50.0%	40.5%	70.4%	<u>71.5%</u>	67.3%	60.0%	41.3%

TABLE III

PERCENTAGE OF TOPOLOGICAL DIFFERENCES DISCOVERED BETWEEN WEIGHTED COMPLEX HIERARCHY MODELS ($s = 0.1$ vs. $s = 0.15$) WITH RANDOM NETWORK ATTACKS

Method	M1	M2	M3
CST	60.21	59.26	68
MST	8.53	8.63	7.37
USP	18.21	19.79	23.79
Weight	40.74	57.89	52.42

TABLE IV

PERCENTAGE OF TOPOLOGICAL DIFFERENCES DISCOVERED BETWEEN WEIGHTED COMPLEX HIERARCHY MODELS ($s = 0.1$ vs. $s = 0.15$) WITH TARGETED NETWORK ATTACKS

Method	M1	M2	M3
CST	85.71	85.24	93.24
MST	61.14	26.86	20.95
USP	31.33	14.95	72.67
Weight	46.10	65.24	59.71

For the random attacks (Fig. 1), even at 50% of connections being attacked, the CST notes an accuracy of 70% (black line). Strictly in terms of robustness, as opposed to best accuracy, the weighted networks prove the best with the least effect noted by increasing randomisation in its topological acuity (green), whereas the CST notes a marked drop off after noise increases beyond 60%. The USP (blue) and MST (red) networks are not at all robust to noise in this scenario with immediate drop offs on the addition of noise.

For the targeted attacks (Fig. 2), the CST network shows the most resilience with no noticeable depreciation of accuracy and, in fact a seeming increase in accuracy, which at first glance is rather surprising. The other methods, in contrast, show a notable decrease in accuracy as more weights are

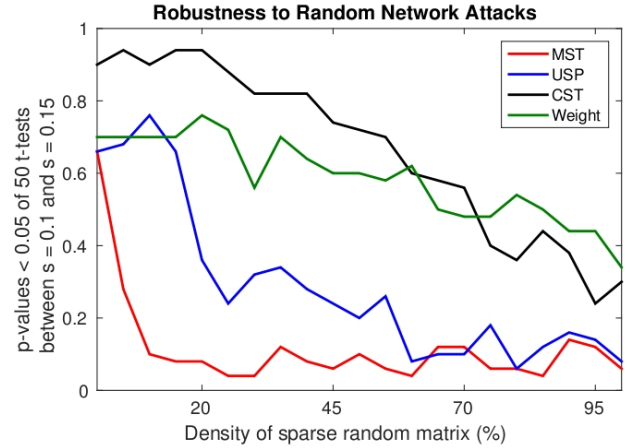


Fig. 1. Plot of percentage accuracy of method for distinguishing topological differences between noisy WCH models against size of random network attacks. The values plotted are the maximum from the three metrics, M1, M2 and M3, for the corresponding technique as indicated in the legend.

randomised. A noticeable exception is in the USP curve (blue) which, again, actually shows an increase in accuracy at around 40-60% of hub weights randomised, after which it drops again towards 100%. We discuss these apparently counter-intuitive slight increases in accuracy in Section IV.

C. Real dataset results

From now on we narrow our focus to comparing non-arbitrary methods since arbitrary approaches are inappropriate for neuroscientific studies where one can pick from an order of $n(n-1)$ thresholds. We can, however, generally state that MST is representative of sparse networks and the CST is

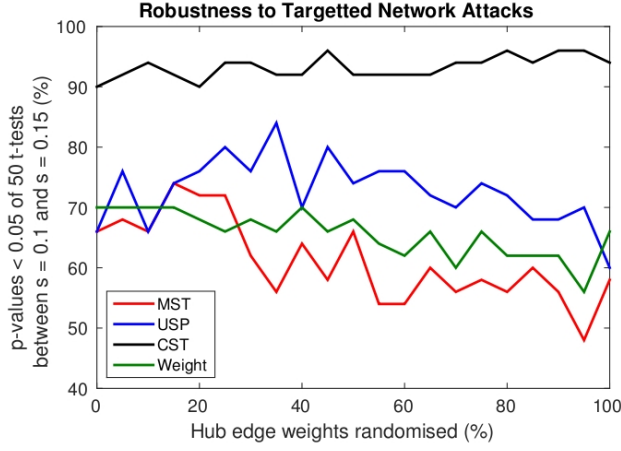


Fig. 2. Plot of percentage accuracy of method for distinguishing topological differences between WCH models against size of hub targeted network attack. The values plotted are the maximum from the three metrics, M1, M2 and M3, for the corresponding technique as indicated in the legend.

representative of medium density networks. The USP density is dependent on the distribution of weights which can vary between datasets.

Table V shows the results for distinguishing the difference in α activity well known to exist between eyes-closed and eyes-open conditions [31]. The CST finds a significant difference in V of eyes open and eyes closed resting state activity indicating that the phase-dependent topology of EEG activity is less scale-free in the eyes-open condition implying greater hub dominance in the eyes-closed condition, see Fig. 4, left. All of the weighted metrics also find significant differences. Neither the MST or USP find any differences between these conditions. Probing further, the weighted metrics in this case are all very highly correlated (all > 0.95 pearson correlation coefficient, ρ , Table VI) within condition. Therefore they cannot be seen to provide any distinct topological information. The corresponding correlations of the CST show a more distinct topological characterisation, see Table VI. Thus, there are clear benefits of the result for the CST beyond the weighted networks in that topological information can be inferred and discussed meaningfully from the results.

Table VII shows the results for distinguishing the difference in β activity existing between Shape and Binding tasks when tested in the Left and Right sides of the screen separately. The only significant difference found for all methods is in V of the CST networks in the Right condition. This indicates that the phase-dependent topology of EEG activity is less scale-free in the Binding condition implying greater hub dominance in the Shape condition, see Fig. 4, right.

Noticeably, the USP completely failed to find meaningful network information in this task because, even after transformation, all the weight magnitudes were in a range such that the shortest weighted path between each pair of nodes was the weight of single edge joining them.

Table VIII shows the results for distinguishing the difference in both α and β activity existing between AD patients and healthy age matched control. Again, the only differences found in testing is for the CST networks. A small effect is noticed

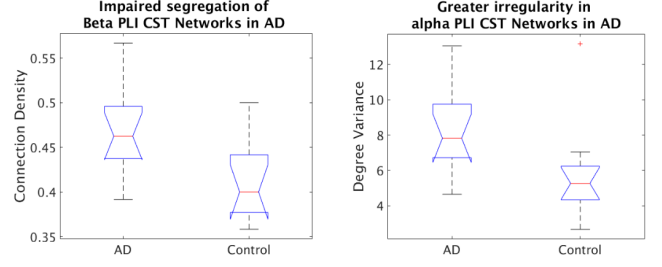


Fig. 3. Box plots of connection density, left, and degree variance, right, for CST networks of AD and control in β and α , respectively.

in V of α activity (Fig. 3, right) and a larger effect is found in the P of β activity (Fig. 3, left). Since P of CST networks is inversely relational to C of arbitrary threshold networks, this tells us that β of AD patients is less segregated than control. Contrasting with this, the activity in α suggest a more scale-free network in the alpha band of AD patients than in age matched control.

TABLE V
TOPOLOGICAL DIFFERENCES DISCOVERED BETWEEN EYES OPEN (EO) AND EYES CLOSED (EC) IN α -BAND 129-CHANNEL EEG PLI NETWORKS. UNDERLINE: BEST VALUE FOR EACH METHOD. BOLD: SIGNIFICANT VALUES.

Method	CST	MST	USP	Weight
M1	0.7504	0.4178	0.5063	0.0016
M2	0.9319	0.4513	0.9942	0.0034
M3	0.0006	0.9616	0.6577	0.0016

TABLE VI
CORRELATIONS OF METRICS IN EYES OPEN (EO) - EYES CLOSED (EC) DATASET FOR THE CST AND WEIGHTED METRICS (WGT)

Metrics	EC (CST)	EO (CST)	EC (wgt)	EO (wgt)
$\rho(M1, M2)$	0.7662	0.9692	0.9993	0.9512
$\rho(M1, M3)$	-0.0458	-0.7301	0.9999	0.9996
$\rho(M2, M3)$	-0.1695	0.6677	0.9994	0.9576

TABLE VII
TOPOLOGICAL DIFFERENCES DISCOVERED BETWEEN SHAPE ONLY AND SHAPE COLOUR BINDING TASKS IN β -BAND 30-CHANNEL EEG PLI NETWORKS. FORMATTING AS IN TABLE V

Method	CST	MST	USP	Weight
Left M1	0.5128	0.7186	-	0.1007
Left M2	0.0898	0.1383	-	0.1010
Left M3	0.8997	0.0911	-	0.1010
Right M1	0.5877	0.1919	-	0.7742
Right M2	0.9196	0.5716	-	0.7733
Right M3	0.0088	0.8146	-	0.7733

TABLE VIII
TOPOLOGICAL DIFFERENCES BETWEEN AD AND CONTROL IN 16-CHANNEL EEG PLI NETWORKS

Method	CST	MST	USP	Weight
α M1	0.0852	0.3468	0.1167	0.6736
α M2	0.3634	0.2630	0.1081	0.4189
α M3	0.0406	0.7324	0.0942	0.5570
β M1	0.0062	0.4618	0.1500	0.7080
β M2	0.0529	0.6245	0.1485	0.4215
β M3	0.1782	0.5437	0.1397	0.5564

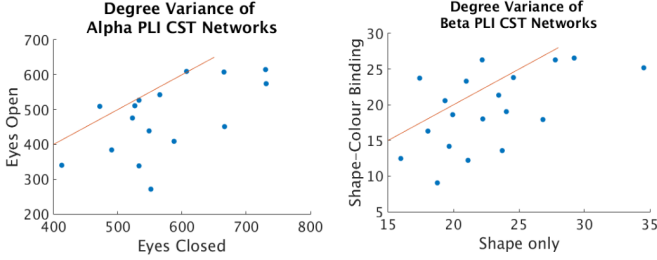


Fig. 4. Scatter plots of degree variance for CST networks of Eyes Open vs Eyes closed resting state conditions in α , left, and degree variance for CST networks of Shape only vs Shape-colour binding conditions in the Right screen in β , right.

IV. DISCUSSION

The CST clearly presents itself as a sensitive and powerful binarisation technique for network modelling of functional connectivity. In simulations it performed to a very high standard in all network sizes and topological comparisons as well as in robustness to network attacks. This was echoed in the results of the real data sets, confirming both the validity of the model for simulations and the validity of the CST as a useful binarisation technique in an array of different neuroscientific situations. From the simulation results we can infer a large part of this ability to the density range in which the CST binarises the network. Thus, the 'one-shot' binarisation of the CST provides a non-arbitrary solution to researchers who need no longer worry about how to select and justify the density at which they threshold in searching for a solution in larger, information rich densities. We must note, of course, that all of our real data was from EEG recordings and thus we are cautious of similar comparisons for e.g. fMRI.

As an important aside, the results show that the random attacks, rather than the targeted attacks, are the most effective at deconstructing the topology of our simulations. This is perhaps counter-intuitive, but may be explained by the fact that only the very top levels of the hierarchy are attacked in this setting, whereas the topology in the remaining levels remains largely intact and, in fact, a new 'top level' emerges, maintaining the differences exhibited in the strength parameter, s , between the two sets of topology. Since these network simulations are found to behave topologically similar to EEG functional networks [20], this provides explanation towards the greater resilience of functional brain networks to targeted attacks [32].

The results for V in both the Eyes open vs closed and Shape vs Binding datasets combined can explain that more intensive stimulation (eyes open and Binding) leads to a drop in network efficiency where more localised activity is required for higher functional processing [1], [3]. The results for the AD dataset indicate both the increased power in binarisation with the CST compared to other approaches and highlights the importance of binarisation itself for distinguishing dysfunctional AD topology.

AD network studies, over varying platforms, network sizes and density ranges, have been found to show seemingly contrasting results [4]. Particularly, Tijm's et al. [4] reported

that these studies were at different density ranges, and in many cases the density range was simply not recorded. Importantly, no functional studies reported density ranges over 25%. Nonetheless, we note that our results are in agreement with a 149 node MEG PLI study by Stam et al. [33], showing lower clustering in AD than control (density not recorded). This is indicative of a move to a more random topology [34].

The results of the simulations confirm the results from the real data and provide interesting information about the effect of density and network size in the ability to discern differences in topology. Table II shows that densities of around 40% appear to be ideal for discerning differences in our simulations while there appears to be a noticeable drop off at 10% density with results around 20 points less than at 20% density. This exemplifies the importance of higher density networks and advises to relax the focus on sparse network techniques in the research community. Since brain functional networks are generally relatively small, calls for computational efficiency in network science need not be heeded here.

In terms of network size our simulations suggest that the larger the networks are, the more likely it is that topological differences will be picked up by commonly used metrics. This trend is bucked by the MST for which there is a marked drop off from 32 nodes to 128 nodes. This, however is easily explained by the fact that at 32 nodes, the MST makes up $2/n = 6.25\%$ of all possible connections whereas at 64 nodes this percentage is 3.12% and for 128 nodes it is just 1.56% which is in line with the previous discussion that lower network densities inhibit the ability to find topological differences.

The MST is seen to be robust to fluctuations of the underlying network [12]. However, in a recent study we argued that the robustness to fluctuations also means a poverty of information, supported by evidence from an EEG dataset of cognitive tasks [17]. This is confirmed in the results shown here where the MST struggles to find differences in simulations and real EEG data. It appears particularly ineffective in larger networks which noticeably corresponds to the MST making up less and less of the connection density as the network grows. It is perhaps no surprise then that the MST performed poorest in comparing 128 node hierarchical networks of $s = 0.15$ vs $s = 0.2$ (8%), $s = 0.2$ vs $s = 0.25$ (4%) and $s = 0.25$ vs $s = 0.3$ (6%).

The USP is the set union of those edges which form the shortest paths between all possible pairs of electrodes. Since, in general, all weights of a functional connectivity network lie between 0 and 1, it is likely that a large percentage of the shortest paths in the network will be constituted of just the single edge joining those nodes. Thus, we can expect very high density networks which only differ in topology by the weakest connections, which would bring spurious results. Indeed, in the original paper [14] the authors did not implement any transformation of the weights and reported densities above 90%. To try and counter this unwanted outcome we used a negative exponential transform of the weights before extracting the union of shortest paths, however, in the end it appeared that this was limited in its ability to mitigate the flaws of this method. This was most apparent in the VSTM tasks where it

turned out that every shortest path was just the edge between each pair of nodes, redundantly returning complete networks. Further work would need to be done regarding the reliability of the USP in order to make it of use to the neuroscience community.

We note that research is also undertaken into statistical methods on expected values of connectivity methods to threshold the network [2]. However, rather than resolving the arbitrary choice problem, it merely diverts it towards the statistical significance paradigm, where arbitrary standards have long been adopted to mitigate an intractable problem. Since, in our case, we can rely on graph topological techniques, the problem is not intractable and there is, as yet, no reason to accept such a defeat. Further problems with this approach relate to difficulties in finding the correct solutions for the numerous new connectivity measures available in a way that is consistent and reliable, biases from the size of available data, and, in the case of data surrogate methods, biases due to network size [2].

V. CONCLUSION

The CST was found to be the most effective technique for distinguishing populations of subtly different network topologies in both simulations and in several relevant real EEG datasets. This overcomes the lack of information generated in very sparse networks such as the MST and avoids complications of weight relations when relying on shortest path algorithms, such as with the USP. The extensive simulation results show that the CST is at least as good if not better than arbitrary density thresholds for distinguishing populations of subtly different topologies, while providing a solution to this arbitrariness itself which is problematic in applied settings. It was also shown to be robust to random and targeted network attacks. In real datasets, the medium density range which the CST occupies does indeed appear to be important for a whole range of neuroscientific questions. Particularly, the CST can help to identify different topologies in resting states, in cognitive tasks and in the clinical setting.

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